Laminar-Nonlaminar Transition for Non-Newtonian Flow in Annuli

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Experimental results on the laminar-nonlaminar transition of aqueous solutions of hydroxyethyl cellulose in axial, isothermal flow in annuli are presented. For each polymer solution the end of the laminar regime is observed to occur at different flow rates at different axial distances from the entrance of the test section. For some polymer solutions in nonlaminar motion, the pressure gradient is not constant over the length of the test section.

These data on transition in annuli are compared with predictions made using Hanks' transition criterion where only viscous contributions are considered and where viscous effects are considered and elastic effects accounted for approximately. Predictions where both viscous and elastic properties of the fluids are considered are in fair agreement with experiments. Predictions using purely viscous theory are less accurate than the viscoelastic predictions.

Experimental measurements of the laminar-nonlaminar transition for non-Newtonian fluids flowing axially in conduits of annular cross section have not been available previously. However, much data in the transition region for non-Newtonian fluids flowing in tubes have been published. Methods for predicting where non-Newtonian fluids flowing in tubes will cease to be laminar have been proposed by several authors. Metzner and Reed (*1*) observed that transition occurs at a nearly constant value of friction factor for a wide class of fluids. Dodge and Metzner (*2*) observed that transition occurred for a gel type of material, Carbopol, at values of the Reynolds number, N'_{Re}, between 2,700 and 3,100. These same authors also observed for aqueous solutions of carboxymethyl cellulose, transition occurred at values of N'_{Re} of slightly greater than 2,100. Meter (*3*) suggested a method for prediction transition in tube flows based on Ellis fluid parameters.

A method for prediction of the laminar-nonlaminar transition which is said to be applicable to fluids of any constitutive equation flowing in conduits of many geometries has been proposed by Hanks (*4*). A similar transition criterion was originally proposed by Ryan and Johnson (*5*) who used their criterion to predict transition for tube flows. Hanks generalized this theory and applied it, with good results, to Newtonian flows in parallel plates, annuli, and recently to rectangular ducts (*6*). Hanks' theory has also been used by Scheele and Greene (*7*) to predict transition in natural convection flows in vertical tubes.

Data given in Table 1 for aqueous solutions of the hydroxyethyl cellulose polymer, Natrosol, flowing in annuli indicate that transitions do not occur at approximately constant values of friction factor or Reynolds number $N_{Re}(n, b)$. Meter's correlation based on Ellis parameters was not successful in correlating the nonlaminar data although the laminar data are described well by the Ellis model (*8*). The remainder of this paper is devoted to testing the usefulness of the Hanks theory in predicting the transition of aqueous hydroxyethyl cellulose solutions in annuli.

THEORY OF HANKS

Hanks (*4*) in generalizing the theory originated by Ryan and Johnson (*5*) defined a stability parameter K:

$$|\rho v \times \nabla \times v| \equiv K |\nabla \cdot \tau| \tag{1}$$

as the ratio of the magnitude of the acceleration term from the equation of motion to the magnitude of the viscous term of the equation of motion. The quantity K is a function of position and at some point, r_c, in many flow fields has a maximum value $\overline{K}$. Hanks postulated that when $\overline{K}$ becomes sufficiently large laminar motion becomes unstable to certain disturbances. The value of $\overline{K}$, where the laminar motion becomes unstable, was determined by Hanks to be 404 when the theory was applied to axial, isothermal, Newtonian flow in tubes. Since the transition criterion does not depend on the constitutive equation or on the geometry of flow, it is thought to be quite generally applicable. However the definition of the criterion as presented by Hanks does not suggest the flow geometries or the types of fluid to which the criterion might be applied.

CALCULATIONS

The equations expressing conservation of momentum for axial, isothermal flow in an annulus, where the velocity is assumed to be $[v_r, v_\theta, v_z] = [0, 0, v_z(r)]$, may be written:

r-component

$$-\frac{\partial p}{\partial r} = \left[\frac{\partial \tau_{rr}}{\partial r} + \frac{1}{r}(\tau_{rr} - \tau_{\theta\theta})\right] \tag{2}$$

z-component

$$-\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(r\ \tau_{rz}) \tag{3}$$

θ-component

$$\frac{\partial p}{\partial \theta} = 0 \tag{4}$$

The velocity profile is given by integration of Equation (3) with the non-Newtonian viscosity defined by $\tau_{rz} = -\eta(dv_z/dr)$

$$v_z = \frac{PR^2}{2}\int_\xi^1 \frac{1}{\eta}\left(\xi - \frac{\lambda^2}{\xi}\right) d\xi \tag{5}$$

where

$$\lambda^2 = \int_b^1 \frac{\xi d\xi}{\eta} \Big/ \int_b^1 \frac{d\xi}{\xi\eta} \tag{6}$$

and $\xi = r/R$

The normal stresses may be obtained by integration of Equation (2).

$$\Pi_{rr}(r, z) = \Pi_{rr}(R, z) - \int_R^r \frac{1}{r}(\tau_{rr} - \tau_{\theta\theta})\, dr \tag{7}$$

$$\Pi_{\theta\theta}(r, z) = \Pi_{rr}(R, z) - (\tau_{rr} - \tau_{\theta\theta}) - \int_R^r \frac{1}{r}(\tau_{rr} - \tau_{\theta\theta})\, dr \tag{8}$$

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Table 1. Experimentally Measured Critical Quantities

	$\left(\frac{PR}{2}\right)_{\text{critical}}$ (dynes/sq.cm.) Segment B or BCD	$\left(\frac{PR}{2}\right)_{\text{critical}}$ (dynes/sq.cm.) Segment CD	$N_{Re}(n, b)$ Segment B or BCD	$N_{Re}(n, b)$ Segment CD	f_{critical} Segment B or BCD	f_{critical} Segment CD
			$b = 0.504$			
0.7% Natrosol-H	3,000	4,000	2,500	4,900	0.0046	0.0078
0.5% Natrosol-H	3,100	2,400	15,700	7,400	0.0023	0.0050
0.3% Natrosol-H	850	1,000	2,300	3,000	0.0067	0.0056
1.0% Natrosol-G	1,800	2,500	1,250	2,032	0.012	0.0064
			$b = 0.250$			
0.7% Natrosol-H	2,200*	—	7,000*	—	0.0026*	—
0.5% Natrosol-H	1,250*	—	7,400*	—	0.0020*	—
0.3% Natrosol-H	490*	—	5,700*	—	0.0029*	—
1.0% Natrosol-G	1,000	650	2,250	1,100	0.0064	0.0135

* Based on pressure measurements over Segment BCD

$$\Pi_{zz}(r, z) = \Pi_{rr}(R, z) + (\tau_{zz} - \tau_{rr}) - \int_R^r \frac{1}{r}(\tau_{rr} - \tau_{\theta\theta})\, dr \quad (9)$$

The stability parameter can be found from Equations (1), (2), and (3) to be:

$$K = 1/2\ \rho \frac{\frac{d}{dr}(v_z^2)}{\left[\left(\frac{\partial p}{\partial z}\right)^2 + \left(\frac{\partial p}{\partial r}\right)^2\right]^{1/2}} \quad (10)$$

The value of $\partial p/\partial z$ will be defined as $-P$.

To calculate $\partial p/\partial r$ the normalization $p = 1/3(\Pi_{rr} + \Pi_{\theta\theta} + \Pi_{zz})$ is used. From the values of the normal stresses given in Equations (7), (8), and (9) the pressure is found by use of the normalization:

$$p(r, z) = 1/3\ [3\pi_{rr}(R, z) - (\tau_{rr} - \tau_{zz})] \quad (11)$$

The controversial assumption, $\tau_{rr} = \tau_{\theta\theta}$, has been made. Then the radial pressure gradient is:

$$\frac{\partial p}{\partial r} = \frac{1}{3}\frac{\partial}{\partial r}(\tau_{zz} - \tau_{rr}) = \frac{1}{3}\frac{\partial}{\partial r}(-\sigma_2) \quad (12)$$

where σ_2 is one of the material functions defined by Coleman and Noll (9). Thus, the stability parameter is found from Equations (10), (5), (6), and (12).

$$K = \frac{\rho\left(\frac{PR}{2}\right)^2 \frac{R^2}{\eta} \left|\left(\xi - \frac{\lambda^2}{\xi}\right)\right| \left|\int_1^\xi \frac{1}{\eta}\left(\xi - \frac{\lambda^2}{\xi}\right) d\xi\right.}{\left[4\left(\frac{PR}{2}\right)^2 + \left[\frac{1}{3}\frac{\partial}{\partial \xi}(\sigma_2)\right]^2\right]^{1/2}} \quad (13)$$

Meter (10) has measured values of σ_2 for Natrosol solu-

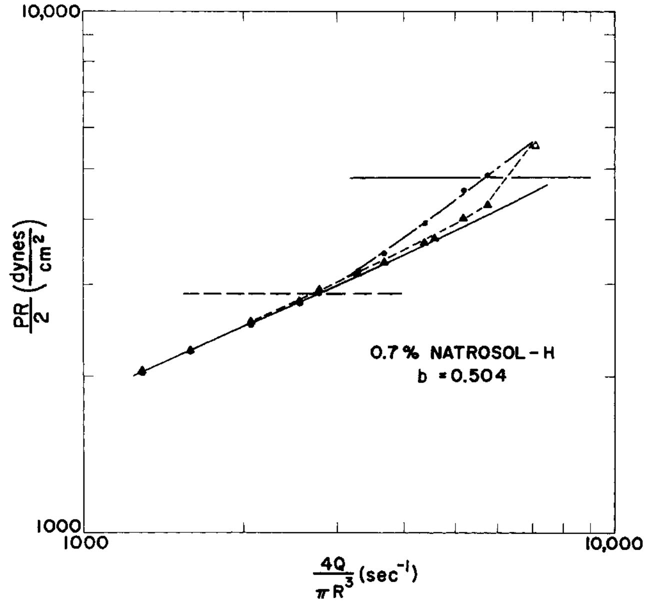

Fig. 1. Flow curves in transition region for aqueous Natrosol solutions flowing in annuli (77°F.). Solid lines = predicted laminar flow curves (8); horizontal solid lines = viscoelastic predictions of $(PR/2)_{\text{critical}}$; horizontal dashed lines = purely viscous predictions of $(PR/2)_{\text{critical}}$; circles = data from segment B or BCD of test section; triangles = data from segment CD. 0.7% Natrosol-H solution, $b = 0.504$.

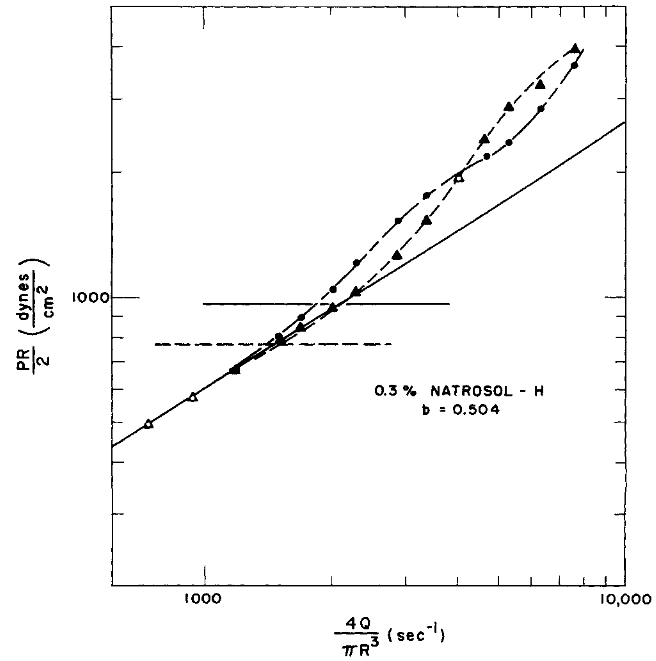

Fig. 2. Flow curves in transition region for aqueous Natrosol solutions flowing in annuli (77°F.). See legend Figure 1. 0.3% Natrosol-H solution, $b = 0.504$.

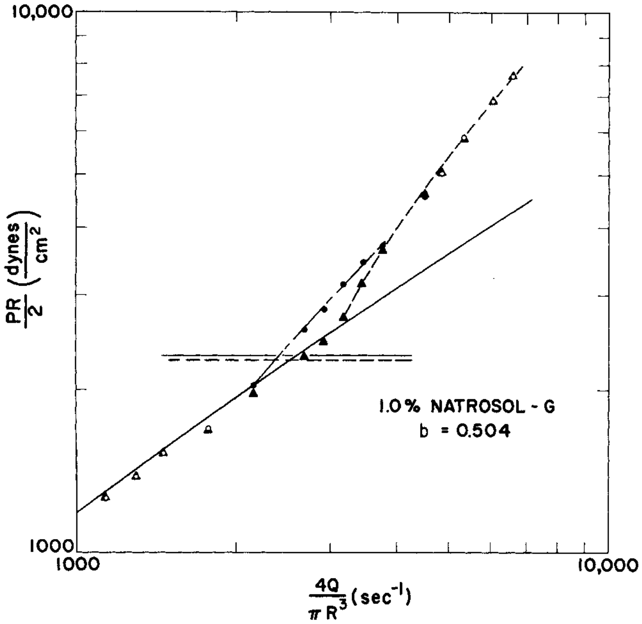

Fig. 3. Flow curves in transition region for aqueous Natrosol solutions flowing in annuli (77°F.). See legend Figure 1. 1.0% Natrosol-G solution, $b = 0.504$.

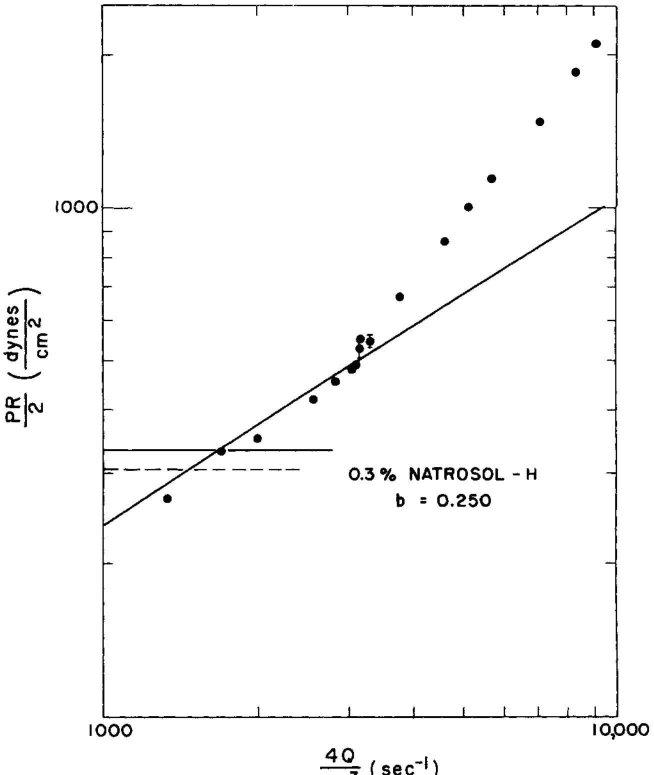

Fig. 4. Flow curves in transition region for aqueous Natrosol solutions flowing in annuli (77°F.). See legend Figure 1. 0.3% Natrosol-H solution, $b = 0.250$.

tions using a cone-plate rheometer. His data may be approximated in cylindrical coordinates by:

$$-\sigma_2 \equiv (\tau_{zz} - \tau_{rr}) = a\tau^s_{rz} \tag{14}$$

Meter reports his normal stress data are accurate to ± 20%. The normal stresses were measured at shearing stresses up to 400 dynes/sq.cm. for the Natrosol-H solutions and to 600 dynes/sq.cm. for the Natrosol-G solutions.

Meter's data indicate that $a = 0.3$ (dynes/sq.cm.)$^{-0.8}$ and $s = 1.8$ for Natrosol-H solutions, and that $a = 1.0$ and $s = 1.0$ for Natrosol-G solutions.

Equations (6), (13), and (14) were used with experimental viscometric and normal stress data to evaluate $\overline{K}$ for various values of $PR/2$. These values of $\overline{K}$ and corresponding values of $PR/2$ were plotted and the value of $PR/2$ where $\overline{K}$ was equal to 404 was designated as the critical value, $(PR/2)_{\text{critical}}$. Similar calculations were made where $\partial p/\partial r$ was put equal to zero to represent the purely viscous fluid. In both calculations viscometric data were entered into the computer in tabular form and interpolations were made between adjacent entries. The normal stress data were represented by Equation (14).

The function, K, shown in Equation (13) has two maxima for the annulus one in the region $b \leq \xi \leq \lambda$ and another in the region $\lambda \leq \xi \leq 1$. For the fluids and annuli studied here the maximum in the region $\lambda \leq \xi \leq 1$ yielded the lower value of $(PR/2)_{\text{critical}}$ for all fluids except the 1.0% Natrosol-G fluids. The maximum yielding the lower value of $(PR/2)_{\text{critical}}$ is considered here to be transition from laminar to nonlaminar flow predicted by the criterion.

Values of $(PR/2)_{\text{critical}}$ for the viscoelastic and purely viscous theory are shown in Tables 2 and 3, respectively.

EXPERIMENTAL DATA

The annulus data were gathered on an ordinary flow loop apparatus (*11*). Annuli with radius ratios, b, of 0.504 and 0.250 were used. The outside tube of both annuli was a 12 ft. long, smooth brass tube with 0.500 in. I.D. The cores were formed by stainless steel tubes with diameters of 0.252 and 0.125 in. Flow rates were measured by weighing fluid collected during a measured time interval. Pressure drops were measured by water-over-mercury and water-over-oil manometers connected to various combinations of 1/16 in. diameter pressure taps drilled through the outside tube. The most upstream pressure tap, tap 1, was 100 outside pipe diameter from the inlet to the test section; tap 2, tap 3, and tap 4 were 150, 175, and 200 outside pipe diameter, respectively, from the inlet to the test section.

The 0.252 in. diameter core was held by four fixed supports placed 0, 24, 264, and 288 outside pipe diameters from the test section inlet. This provided an unobstructed entrance length between the core supports and the first pressure tap of 152 equivalent diameters* and an exit length between the last pressure tap and nearest support of 128 equivalent diameters. The 0.125 in. diameter core was centered by two supports, one placed at the inlet of the test section and one placed at the outlet of the test section. This provided an unobstructed entrance length of 133 equivalent diameters and an exit length of 117 equivalent diameters. Tension was held on both cores to reduce sagging of the cores between the supports.

The cores were checked for eccentricity by using sucrose solutions moving in laminar motion through the annuli and comparing the data with Lamb's solution, equation (2.4-16) in reference *12*. These experiments agreed with Lamb's solution to within 0.8% for the annulus with $b = 0.504$ and to within 4.8% for the annulus with $b = 0.250$.

The pressure taps were tested for uniformity by using water in turbulent flow in the open tube, water in turbulent flow in both annuli, and sucrose solutions in laminar flow in both annuli. All combinations of pressure taps used here measured pressure gradients at the same flow rate which were in agreement to better than 1% in all cases. Pressure drops for three tap combinations are reported here:

Segment B — tap 1 and tap 2
Segment CD — tap 2 and tap 4
Segment BCD — tap 1 and tap 4

* Equivalent diameter is defined as $2R\,(1-b)$.

TABLE 2. COMPARISON OF EXPERIMENT WITH THEORETICAL PREDICTIONS VISCOELASTIC THEORY

Fluid	$b \leq \frac{r}{R} \leq \lambda$: $\frac{r_c}{R}$	$b \leq \frac{r}{R} \leq \lambda$: $\left(\frac{PR}{2}\right)_{\text{critical}}$ $\left(\frac{\text{dynes}}{\text{sq.cm.}}\right)$	$b \leq \frac{r}{R} \leq \lambda$: % Error in $\left(\frac{PR}{2}\right)_{\text{critical}}$	$\lambda \leq \frac{r}{R} \leq 1$: $\frac{r_c}{R}$	$\lambda \leq \frac{r}{R} \leq 1$: $\left(\frac{PR}{2}\right)_{\text{critical}}$ $\left(\frac{\text{dynes}}{\text{sq.cm.}}\right)$	$\lambda \leq \frac{r}{R} \leq 1$: % Error in $\left(\frac{PR}{2}\right)_{\text{critical}}$	Experimental Values $\left(\frac{PR}{2}\right)_{\text{critical}}$ $\left(\frac{\text{dynes}}{\text{sq.cm.}}\right)$
				$b = 0.504$			
0.7% Natrosol-H	0.588	5,650	+41	0.905	4,820	+21	4,000
0.5% Natrosol-H	0.592	3,170	+26	0.907	2,730	+12	2,400
0.3% Natrosol-H	0.598	1,090	+ 9	0.898	970	− 3	1,000
1.0% Natrosol-G	0.572	2,150	−14	0.909	2,320	− 7	2,500
				$b = 0.250$			
0.7% Natrosol-H	0.360	2,770	+26	0.861	2,170	− 2	2,200
0.5% Natrosol-H	0.364	1,200	− 4	0.860	950	−24	1,250
0.3% Natrosol-H	0.379	380	−22	0.836	335	−32	490
1.0% Natrosol-G	0.337	760	−24	0.839	870	−13	1,000

During experiments with each fluid no pattern for selecting order of flow rates was used.

The end of the laminar region was judged to be where the values of $PR/2$ measured from pressure drop readings in segment CD began to deviate sharply from the laminar values predicted by the viscometric data. When pressure drop readings from segment CD were not available pressure readings over segment BCD were used to determine the end of the laminar region. The experimental critical values of quantities shown in Tables 1 and 2 are not more than $\pm$ 10% accurate.

For the following fluids flowing in the annulus with $b = 0.504$ the pressure gradients in segment B began to deviate from laminar values at lower flow rates than pressure gradient measurements in segment CD: 0.7 and 0.3% Natrosol-H, and 1.0% Natrosol-G. It appears in these cases that disturbances introduced at the inlet were not damped out in the test section entrance length. The other two fluids for which pressure measurements in both segments B and CD are available, the 0.5% Natrosol-H used in the annulus with $b = 0.504$ and the 1.0% Natrosol-G solution in the annulus with $b = 0.250$ showed that the pressure gradient measured over segment CD began to deviate from laminar values at lower flow rates than pressure gradients measured in segment B. These two cases are consistent with the results of the calculation of Hahneman, Freeman, and Finston (*13*) who showed, for Newtonian fluids in tubes, the value of the critical Reynolds number decreases from high values at the entrance to 2,100 farther downstream.

It is interesting to note that Dodge (*14*) observed quite similar unequal pressure gradient measurements in his tube flow experiments with carboxymethyl cellulose solutions. Other investigators have apparently not reported pressure gradients over more than one pipe section at the same flow rate for nonlaminar motion.

Neither of the values of $(PR/2)_{\text{critical}}$ predicted from the two maxima of Equation (13) appear to correspond to the lower values of $(PR/2)_{\text{critical}}$ measured experimentally. The lower experimental value appears to be an entrance effect rather than a transition from laminar to nonlaminar motion. No consistent treatment of these unequal pressure gradients will be given at this time.

DISCUSSION

Figures 1 through 5 present some of the experimental data in the region of transition from laminar to nonlaminar motion. The laminar flow curves were predicted from viscometric data by a method described previously (*8*) are shown as a solid line. Values of $(PR/2)_{\text{critical}}$ predicted from the maximum using viscometric and normal stress data in Equation (13) are shown as solid horizontal lines. Predictions using purely viscous theory are shown by dashed horizontal lines.

Table 2 shows values of $(PR/2)_{\text{critical}}$ predicted by using viscometric and normal stress data in Equation (13). Values of $(PR/2)_{\text{critical}}$ in both the interval $b \leq \xi \leq \lambda$ (near the center tube) and $\lambda \leq \xi \leq 1$ (near the outside tube) are shown. The maximum located near the outside tube yields the lower value of $(PR/2)_{\text{critical}}$ for all fluids except the 1.0% Natrosol-G. A maximum error of 32% is seen in the predictions, and the average error of the eight cases is 14%.

Table 3 shows values of $(PR/2)_{\text{critical}}$ using only viscous effects. Again the maximum near the outside tube yields the lower value of $(PR/2)_{\text{critical}}$ for all fluids except the 1.0% Natrosol-G solutions. The purely viscous theory pre-

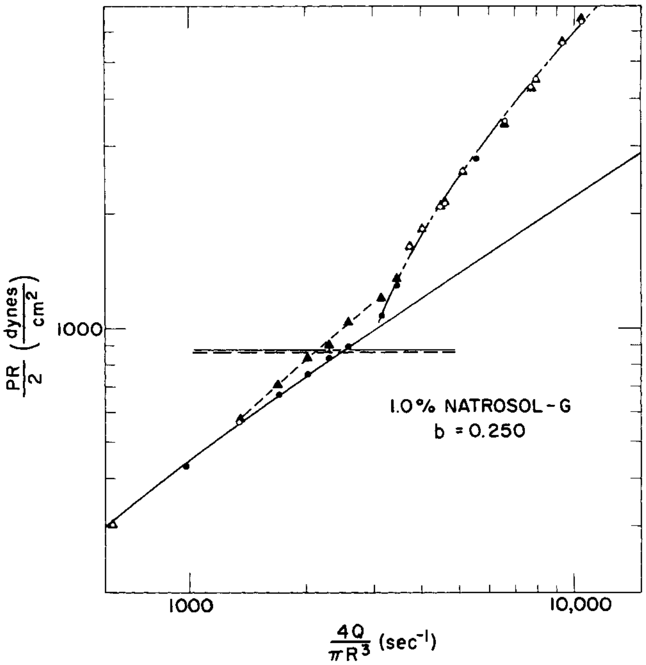

Fig. 5. Flow curves in transition region for aqueous Natrosol solutions flowing in annuli (77°F.). See legend Figure 1. 1.0% Natrosol-G solution, $b = 0.250$.

TABLE 3. COMPARISON OF EXPERIMENT WITH THEORETICAL PREDICTIONS
PURELY VISCOUS THEORY

Fluid	$b \leq \frac{r}{R} \leq \lambda$: $\frac{r_c}{R}$	$b \leq \frac{r}{R} \leq \lambda$: $\left(\frac{PR}{2}\right)_{\text{critical}}$ $\left(\frac{\text{dynes}}{\text{sq.cm.}}\right)$	$b \leq \frac{r}{R} \leq \lambda$: % Error in $\left(\frac{PR}{2}\right)_{\text{critical}}$	$\lambda \leq \frac{r}{R} \leq 1$: $\frac{r_c}{R}$	$\lambda \leq \frac{r}{R} \leq 1$: $\left(\frac{PR}{2}\right)_{\text{critical}}$ $\left(\frac{\text{dynes}}{\text{sq.cm.}}\right)$	$\lambda \leq \frac{r}{R} \leq 1$: % Error in $\left(\frac{PR}{2}\right)_{\text{critical}}$	Experimental Values $\left(\frac{PR}{2}\right)_{\text{critical}}$ $\left(\frac{\text{dynes}}{\text{sq.cm.}}\right)$
				$b = 0.504$			
0.7% Natrosol-H	0.553	2,640	—37	0.931	2,885	—27	4,000
0.5% Natrosol-H	0.558	1,650	—31	0.924	1,800	—25	2,400
0.3% Natrosol-H	0.566	700	—30	0.911	770	—23	1,000
1.0% Natrosol-G	0.570	2,060	—18	0.907	2,280	—13	2,500
				$b = 0.250$			
0.7% Natrosol-H	0.295	1,340	—39	0.890	1,590	—28	2,200
0.5% Natrosol-H	0.306	640	—49	0.871	770	—38	1,250
0.3% Natrosol-H	0.323	250	—49	0.841	305	—38	490
1.0% Natrosol-G	0.330	690	—31	0.841	860	—14	1,000

diction are from 18 to 38% lower than experimental data.

It can be concluded that Hanks' transition criterion is useful in predicting the transition from laminar to nonlaminar motion for non-Newtonian fluids flowing in annuli. There is some indication that the accuracy of the predictions is increased if elastic effects are considered in evaluating the transition criterion. The elastic effects have been accounted for here only approximately in evaluating the transition criterion. The Weissenberg hypothesis has been made and data on the first normal stress have been extrapolated to higher values of shear stress.

Entrance effects in the transition and nonlaminar region were observed although a minimum of 133 equivalent diameters entrance length was allowed. It is suggested that experimenters look for these entrance effects and unequal pressure gradients in the nonlaminar region in subsequent nonlaminar experiments with fluids which manifest elastic behavior. Perhaps a systematic study will lead to better understanding of nonlaminar motion of viscoelastic fluids.

ACKNOWLEDGMENT

The author wishes to thank Professor R. B. Bird, Dr. D. M. Meter, and Professor R. W. Hanks for their help, to the Wisconsin Alumni Research Foundation, Shell Oil Company, and the National Science Foundation Grant G-11996 for financial support, Los Alamos Scientific Laboratory and Gulf General Atomic for making available their computing facilities and assistance in preparation of this manuscript.

NOTATION

- a = defined by Equation (14)
- b = radius ratio, ratio of outside radius of inside tube to inside radius of outside tube of annulus, $0 \leq b \leq 1$
- f = friction factor = $PR/\rho v^2 (1-b)$
- K = stability parameter defined in Equation (1)
- m = power law parameter
- N'_{Re} = Reynolds number defined by Metzner and Reed (*1*)
- $N_{Re}(n, b)$ = power law Reynolds number for annulus defined by (*15*)
- n = power law model parameter defined for axial annular flow by $\tau_{rz} = -m|dv_z/dr|^{n-1}\, dv_z/dr$
- p = pressure ≡ 1/3 trace (π)
- P = $-\partial p/\partial z$
- Q = volume flow rate
- R = inside radius of outside tube of annulus
- r = radial coordinate
- r_c = value of radius at which K assumes a maximum
- s = defined by Equation (14)
- v = average velocity
- v_i = component of velocity in ith direction
- z = axial coordinate

Greek Letters

- δ_{ij} = Kronecker delta
- η = non-Newtonian viscosity
- θ = angular coordinate
- λ = value of ξ in annulus where shear stress, τ_{rz}, is zero
- ξ = r/R
- π_{ij} = stress tensor = $\tau_{ij} + \delta_{ij}\, p$
- ρ = fluid density
- σ_2 = material function defined by Equation (14)
- τ_{ij} = shear stress tensor

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Manuscript received March 31, 1967; revision received May 2, 1968; paper accepted May 5, 1968. Paper presented at AIChE Salt Lake City meeting.